**Exam 2**
**Sections 2.1-2.5 and 3.1-3.4**

**Name:**_____

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Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications.

**Show your work!**

**1.** (10 points) Draw the phase diagram for the autonomous differential equation

$$\frac{dx}{dt} = (x^2 - 5x + 4)(x^2 - 16)$$

and determine which critical points are stable and unstable.

**2.** (10 points) Consider a rabbit population satisfying the logistic equation

$$\frac{dP}{dt} = 2P - (0.005)P^2,$$

where $t$ is measured in years. If the initial population is 700 rabbits, how many months does it take for $P(t)$ to reach 105% of its limiting population $M$?

**3.** (3 points) Recall that an object's velocity (moving vertically) is given by

$$\frac{dv}{dt} = -g - \rho v^p,$$

where $g$ is the force of gravity, $\rho = \frac{k}{m} > 0$, and $1 \leq p \leq 2$. Suppose a team of scientists are trying to determine a projectile's escape velocity from Earth's atmosphere. That team of scientists makes the assumption that $p = 2$ and finds that the initial velocity required to escape Earth's atmosphere (without additional thrust) is given by

$$v_0 = \sqrt{\frac{2GM}{R}}$$

where $M$ is the mass of the Earth and $R$ is its equatorial radius. Give a sentence of justification as to why this initial velocity will be sufficient to escape Earth's atmosphere for all values of $p$.

**4.** (7 points) Consider a body that moves horizontally through a medium whose resistance is given by

$$\frac{dv}{dt} = -2v^{3/2}.$$

Assuming that $v(0) = 1$ and $x(0) = 1$, find the position $x(t)$ as a function of $t$.

**5.** (10 points) Find the general solution of the differential equation

$$6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4 = 0$$

which has characteristic function

$$(r^2 + 4)(6r^2 + 5r + 1) = 0.$$

**6.** (10 points) An 8-lb weight (mass $m$=0.25 slugs) is attached both to a vertically suspended spring that it stretches 12 in. and to a dashpot that provides 2 lb of resistance for every foot per second of velocity.

(a) The weight is pushed up 6 in above its static equilibrium position and then released from rest at time $t = 0$, find its position function $x(t)$.

(b) Determine if the motion is over-damped, critically damped or under-damped.

**Hint:** If you can not figure out the constants, make a guess and do the rest of the problem to demonstrate your ability to do other aspects of the problem for partial credit.